All work must be shown to be awarded full credit.
Provide exact solutions to all problems, unless otherwise stated. A scientific calculator is allowed.

Student Name: ____________________ **ID:**__________________________
Instructor: ______________________

Exam Score: ________

1) Determine whether the Mean Value Theorem can be applied to f on the closed interval $[a, b]$. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

$$f(x) = \frac{x-2}{x}, \quad [1, 3]$$

M.V.T can be applied – f is continuous and differentiable on the given interval

$f(x) = 1 - \frac{2}{x} = 1 - 2x^{-1}$

$f'(x) = 2x^{-2} = \frac{2}{x^2}$

$\frac{f(3)-f(1)}{3-1} = \frac{1/3 + 1}{2} = \frac{4/3}{2} = \frac{2}{3}$

$\frac{2}{x^2} = \frac{2}{3} \rightarrow x^2 = 3 \rightarrow x = \sqrt{3}$

$\boxed{c = \sqrt{3}}$ $(\simeq 1.732)$

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2) Let $f(x) = \frac{2x}{x^2+1}$ be a function defined on its domain.
   a) Find the intervals on which f is increasing and/or decreasing.
   b) Classify the relative extrema of f.

$$f'(x) = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2}$$

$$-2x^2+2=0 \to x^2=1 \to x=1,\ x=-1$$

a)

	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
T.V.	$x=-2$	$x=0$	$x=2$
Sign of f'	$-$	$+$	$-$
Conclusion	$\downarrow$	$\uparrow$	$\downarrow$

f is decreasing on $(-\infty,-1)\cup(1,\infty)$

f is increasing on $(-1,1)$

b) $f(-1)=-1$; rel. min. at $(-1,-1)$

$f(1)=1$; rel. max. at $(1,1)$

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3) Consider $f(x) = x^4 + 2x^3 - 12x^2 + 2x + 4$
   a) Determine the inflection points of f.
   b) State the intervals of concavity for f.

$$f'(x) = 4x^3 + 6x^2 - 24x + 2$$

$$f''(x) = 12x^2 + 12x - 24 = 12(x^2 + x - 2)$$

$$12(x+2)(x-1) = 0 \rightarrow x = -2, \; x = 1$$

possible inflection pts

	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
T.V.	$x = -3$	$x = 0$	$x = 2$
sign of f''	+	−	+
	C.U.	C.D.	C.U.

a) $f(-2) = -48$, $f(1) = -3$

inflection points at $(-2, -48)$ and $(1, -3)$

b) concave up on $(-\infty, -2) \cup (1, \infty)$

concave down on $(-2, 1)$

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4) Find the limits algebraically.

a) $\lim_{x\to\infty} \frac{3x+1}{\sqrt{9x^2+1}}$

(✱) for $x>0$, $x=\sqrt{x^2}$

$$\lim_{x\to\infty} \frac{(3x+1)\left(\frac{1}{x}\right)}{\sqrt{9x^2+1}\left(\frac{1}{x}\right)} = \lim_{x\to\infty} \frac{3+\frac{1}{x}}{\frac{\sqrt{9x^2+1}}{x}} = \lim_{x\to\infty} \frac{3+\frac{1}{x}}{\frac{\sqrt{9x^2+1}}{\sqrt{x^2}}}$$

$$= \lim_{x\to\infty} \frac{3+\frac{1}{x}}{\sqrt{\frac{9x^2+1}{x^2}}} = \lim_{x\to\infty} \frac{3+\overbrace{\frac{1}{x}}^{\to 0}}{\sqrt{9+\underbrace{\frac{1}{x^2}}_{\to 0}}} = \frac{3}{\sqrt{9}} = \frac{3}{3} = 1$$

b) $\lim_{x\to-\infty} \frac{3x+1}{\sqrt{9x^2+1}}$ all steps identical except (✱),

where if $x<0$, $x=-\sqrt{x^2}$

So $\lim_{x\to-\infty} \frac{3x+1}{\sqrt{9x^2+1}} = -1$

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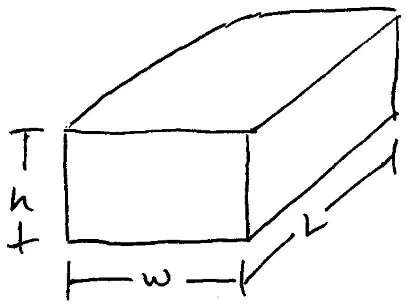
5) Determine the dimensions that will maximize the volume of a rectangular solid with a base whose length is twice the width, and has a surface area of 972 cm^2.



Maximize Volume

$V = Lwh$

Secondary

$L = 2w$

and

$2Lw + 2Lh + 2wh = 972$

$2(2w)w + 2(2w)h + 2wh = 972$

$4w^2 + 6wh = 972$

$6wh = 972 - 4w^2$

$h = \dfrac{972 - 4w^2}{6w}$

$V = 2w^2h$

$V(w) = 2w^2\left(\dfrac{972 - 4w^2}{6w}\right)$

$= \frac{1}{3}w(972 - 4w^2)$

$= 324w - \frac{4}{3}w^3$

$V'(w) = 324 - 4w^2$

$-4w^2 + 324 = 0$

$4w^2 = 324$

$w^2 = 81$

$w = 9$, $h = \dfrac{972 - 4(9)^2}{6(9)} = 12$

$L = 2w = 18$

9 cm × 18 cm × 12 cm